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Popular Summary:

Dynamo models for the Sun have become increasingly more sophisticated but cannot explain the 22-year magnetic cycle. An essential element appears to be missing. We proposed (Mayr, Wolff, Hartle, *GRL*, 28, 2001) that the periodicity of the solar magnetic cycle is determined by wave mean flow interactions analogous to those driving the Quasi Biennial Oscillation in the Earth's atmosphere. Here we discuss some properties of the kinematic dynamo that would generate the magnetic fields. The toroidal magnetic field is directly driven by zonal flow and is relatively large in the source region. Consistent with observations, this field peaks at low latitudes and has opposite polarities in both hemispheres. The oscillating poloidal magnetic field component is driven by the meridional circulation. Relative to the toroidal and the time independent field components, the poloidal magnetic field is small in dynamo region but is expected to grow as it is transported to the surface. Since the meridional and zonal flow oscillations are out of phase, the poloidal magnetic field peaks during times when the toroidal field reverses direction, which is observed. With the proposed wave driven flow oscillation, the magnitude of the oscillating poloidal magnetic field increases with the mean rotation rate of the fluid. This is consistent with the Bode-Blackett empirical scaling law, which reveals that in massive astrophysical bodies the magnetic moment tends to increase with the angular momentum of the fluid.

Abstract: We have proposed (Mayr, Wolff, Hartle, *GRL*, 28, 2001) that the periodicity of the solar magnetic cycle is determined by wave mean flow interactions analogous to those driving the Quasi Biennial Oscillation in the Earth's atmosphere. Upward propagating gravity waves would produce oscillating flows near the top of the radiation zone that in turn would drive a kinematic dynamo to generate the 22-year solar magnetic cycle. The dynamo we propose is built on a given time independent magnetic field \mathbf{B} , which allows us to estimate the time dependent, oscillating components of the magnetic field, $\Delta\mathbf{B}$. The toroidal magnetic field, ΔB_ϕ , is directly driven by zonal flow and is relatively large in the source region, $\Delta B_\phi/B_\theta \gg 1$. Consistent with observations, this field peaks at low latitudes and has opposite polarities in both hemispheres. The oscillating poloidal magnetic field component, ΔB_θ , is driven by the meridional circulation, which is difficult to assess without a numerical model that properly accounts for the solar atmosphere dynamics. Scale-analysis suggests that ΔB_θ is small compared to B_θ in the dynamo region. Relative to B_θ , however, the oscillating magnetic field perturbations are expected to be transported more rapidly upwards in the convection zone to the solar surface. As a result, ΔB_θ (and ΔB_ϕ) should grow relative to B_θ , so that the magnetic fields reverse at the surface as observed. Since the meridional and zonal flow oscillations are out of phase, the poloidal magnetic field peaks during times when the toroidal field reverses direction, which is observed. With the proposed wave driven flow oscillation, the magnitude of the oscillating poloidal magnetic field increases with the mean rotation rate of the fluid. This is consistent with the Bode-Blackett empirical scaling law, which reveals that in massive astrophysical bodies the magnetic moment tends to increase with the angular momentum of the fluid.

1. Introduction

Conventional dynamo models for the Sun have become increasingly more sophisticated. These models, however, suffer because they cannot explain the 22-year magnetic cycle. An essential element appears to be missing.

Earlier models that placed the dynamo in the convection region failed because buoyancy expelled the magnetic field at a rate that was unacceptable. More recent models and helioseismic observations suggest that the dynamo is seated near or below the base of the convection region,

where the magnetic buoyancy is sufficiently small so that magnetic fields with sufficient magnitude can be generated (e.g., Gilman, 2000). In this region of the Sun, the plasma pressure is so large ($\beta \gg 1$) that magnetic fields cannot influence the dynamics significantly. Magneto hydrodynamic dynamos, in which magnetic fields could counteract the flow, are not effective to produce the periodic 22-year oscillation. The dynamo instead must be kinematic (e.g., Chouduri, 2000), and for that an oscillating flow is required to generate the magnetic cycle. Such an oscillating flow has been recently proposed by Mayr et al. (2001), and the underlying fluid dynamics has been elaborated by Mayr and Wolff (2001).

The basic mechanism proposed to drive a solar flow oscillation with periodicity of 22 years has been studied extensively in the Earth's atmosphere and is well understood. As was demonstrated in the seminal papers of Lindzen and Holton (1968) and Holton and Lindzen (1972), upward propagating waves, planetary waves in their case, can accelerate the zonal flow at low latitudes to produce a persistent periodic oscillation with a period around 24 months that describes the observed Quasi Biennial Oscillation (QBO). While the QBO is influenced by the seasonal variations of solar heating, Lindzen and Holton established that in principle such a flow oscillation can be generated by a steady flow of upward propagating waves without external time dependent forcing.

This mechanism requires gravity waves that propagate upwards. The proposed solar dynamo is therefore placed in the radiative regime just below the convection region, where the dynamical condition of low convective stability is well suited for wave mean flow interactions. The low convective stability in this region helps to produce the desired oscillation period of 22 years, which otherwise would be much longer owing to the large length scales involved in the dynamics of the Sun. Employing reasonable wave amplitudes in a parameterization (Hines, 1997a, b) that was successfully applied to describe the terrestrial QBO, a simplified analytic model produced oscillating zonal flow velocities of about 20 m/s at the solar equator (Mayr et al., 2001). With the Coriolis force vanishing at the equator, such a flow can be derived simply from a 1D solution of the zonal momentum equation without considering the remaining equations for conservation of energy, mass, and momentum. In 2D modeling studies that apply a globally uniform wave source to the terrestrial atmosphere, the zonal flow oscillation is shown to be confined to low latitudes. This is an important property of wave driven flow oscillations, and it is caused by the meridional circulation that redistributes momentum away from the equator.

The essential properties of this flow oscillation, such as its periodicity and the amplitudes of the individual flow velocities, are all determined by the wave driven fluid dynamics without involving the magnetic field. The flow oscillation so derived, with a 22-year cycle, would then drive a kinematic dynamo to generate a magnetic field oscillation with the same periodicity. In the following, we discuss the general properties of this dynamo.

2. Kinematic Dynamo Generated by Flow Oscillation

The flow oscillation to generate the dynamo would occur in the radiative interior of the Sun just below the convective envelop. This region is convectively stable so that gravity waves can propagate, and there is some observational evidence for these waves (Wolff, 1983) although they are difficult to detect (Palle, 1991). In Figure 1a we show the radial distribution of the buoyancy frequency, N , taken from a standard solar model. This reveals N decreasing rapidly towards the base of the convection region, which favors wave interactions with the background flow. Solar models account for radiative transfer but not dynamics, and the distribution of N is therefore uncertain as indicated with the dashed portion in Figure 1a.

Helioseismology now provides information about the differential rotation inside the Sun. In Figure 1b we show the best-fit solar rotation profiles of Charbonneau, et al. (1999). The differential rotation of the convective envelope makes the transition to the solid body rotation of the radiative interior through the tachocline. This reveals that the abatement of differential rotation extends much deeper into the interior at low latitudes than at high latitudes, although the rotation rate (relative to the interior) is much smaller near the equator. Fritts et al. (1998) proposed that waves originating in the convection region could accelerate such a flow, and it is understandable that it would be confined to equatorial latitudes where momentum is not redistributed efficiently by the meridional circulation. And in a similar vane, the wave driven flow oscillation we propose would also be confined to low latitudes (Mayr et al., 2001; Mayr and Wolff, 2001).

In Figure 1c, we illustrate what our proposed flow oscillation is like when generated by waves propagating out of the solar interior. Here we show a snapshot of the flow with peak velocity of 20 m/s and weaker reversing flow at a deeper level. The wave mechanism would

cause this flow pattern to migrate slowly down on a time scale of 22 years. A steady flow of waves would continually regenerate this reversing flow analogous to the Quasi Biennial Oscillation (QBO) in the terrestrial atmosphere (see Lindzen and Holton, 1968). The dashed curves in Figure 1b illustrate how such a flow would modulate solar differential rotation during extremes in the solar cycle.

In the dynamo region of the Sun, we delineate the magnetic field, $\mathbf{B}(t) = \mathbf{B} + \Delta\mathbf{B}(\omega t)$, into time independent and time dependent components, with $\omega = 2\pi/22$ years. Considering the electrical conductivity of the solar interior, the decay time of the magnetic field is so long that \mathbf{B} may in part be the remnant of a fossil field. Whatever generates this magnetic field, we shall assume here that it exists and examine the time variations in $\Delta\mathbf{B}(\omega t)$ as driven by the flow oscillation.

The kinematic dynamo discussed here can be understood, to first order, in the framework of a 2D model. We emphasize, however, that the 2D formulation for our model is a simplification that should be understood in the spirit of Ackhom's razor. A 3D model would certainly be more realistic, but in our case, we believe, that is not essential.

We consider the kinematic dynamo equation

$$\frac{\partial}{\partial t} \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad (1)$$

where η is the magnetic diffusivity whose scale time is taken to be long relative to the 22-year cycle time in the Sun. Consequently, we neglect the corresponding term in Eq. (1). Under this condition, the linearized dynamo equation for the time dependent component of the magnetic field yields

$$\frac{\partial}{\partial t} \Delta\mathbf{B} = \nabla \times (\mathbf{V} \times \Delta\mathbf{B}) + \nabla \times (\Delta\mathbf{V} \times \mathbf{B}) \quad (2)$$

with the flow, $\mathbf{V}(t) = \mathbf{V} + \Delta\mathbf{V}(\omega t)$, delineated (like \mathbf{B}) into time independent and time dependent components, respectively. In 2D, Eq. (2) yields the following equations for the magnetic field components,

$$\begin{aligned}
i\omega \Delta B_\phi - \left[\frac{1}{r} \frac{\partial}{\partial \theta} V_\phi \Delta B_\theta \right] + \left\{ \frac{1}{r} \frac{\partial}{\partial \theta} V_\theta \Delta B_\phi \right\} - \left\{ \frac{\partial}{\partial r} (V_\phi \Delta B_r - V_r \Delta B_\phi) \right\} \approx \\
\approx \frac{1}{r} \frac{\partial}{\partial \theta} \Delta V_\phi B_\theta - \left\{ \frac{1}{r} \frac{\partial}{\partial \theta} \Delta V_\theta B_\phi \right\} + \left\{ \frac{\partial}{\partial r} (\Delta V_\phi B_r - \Delta V_r B_\phi) \right\}
\end{aligned} \quad (3)$$

$$i\omega \Delta B_\theta + \left[\frac{\partial}{\partial r} V_r \Delta B_\theta \right] - \left\{ \frac{\partial}{\partial r} V_\theta \Delta B_r \right\} \approx -\frac{\partial}{\partial r} \Delta V_r B_\theta + \left\{ \frac{\partial}{\partial r} \Delta V_\theta B_r \right\} \quad (4)$$

$$i\omega \Delta B_r - \left[\frac{1}{r} \frac{\partial}{\partial \theta} V_r \Delta B_\theta \right] + \left\{ \frac{1}{r} \frac{\partial}{\partial \theta} V_\theta \Delta B_r \right\} \approx \frac{1}{r} \frac{\partial}{\partial \theta} \Delta V_r B_\theta - \left\{ \frac{1}{r} \frac{\partial}{\partial \theta} \Delta V_\theta B_r \right\} \quad (5)$$

where some of the terms, identified with brackets, will be eliminated in our simplified analysis, and r , θ , and ϕ , are respectively the radial, meridional, and zonal (azimuthal) directions.

Given the time independent components of the magnetic field, B_ϕ , B_θ , B_r , and velocities, V_ϕ , V_θ , V_r , the oscillating flows, $\Delta V_\phi(\omega t)$, $\Delta V_\theta(\omega t)$, $\Delta V_r(\omega t)$, generate in Eqs. (3) through (5) the oscillating magnetic field components $\Delta B_\phi(\omega t)$, $\Delta B_\theta(\omega t)$, $\Delta B_r(\omega t)$. Equation (3) describes the toroidal magnetic field, and Eqs. (4) and (5) describe the poloidal field, the latter being decoupled from the former.

With a simplified analytic model (Mayr et al., 2001), we evaluated $\Delta V_\phi(\omega t)$ at the equator where the Coriolis force vanishes, decoupling the zonal and meridional momentum balances. However, $\Delta V_r(\omega t)$ and $\Delta V_\theta(\omega t)$ cannot be treated without involving the full set of conservation equations in a 2D (or 3D) model. Notwithstanding this difficulty, we shall attempt to explore here some of the properties of the proposed dynamo.

With $\text{div}(\mathbf{V}, \Delta \mathbf{V}) \approx 0$, $\text{div}(\mathbf{B}, \Delta \mathbf{B}) = 0$, $\partial/\partial r \gg (1/r)\partial/\partial \theta$, and $V_\phi > V_\theta$, in the oscillating flow region, we may assume that in general $(V_\phi, \Delta V_\phi) > (V_\theta, \Delta V_\theta) > (V_r, \Delta V_r)$, and $(B_\phi, \Delta B_\phi) > (B_\theta, \Delta B_\theta) > (B_r, \Delta B_r)$. Considering the known properties of the flow oscillation and resulting magnetic fields, we can simplify the equations by eliminating the terms in curly brackets so that the system becomes more tractable. It turns out that Eq. (4) is then exact at the equator, where $(B_r, \Delta B_r)$, $(V_\theta, \Delta V_\theta)$ vanish.

3. Magnetic Fields in the Dynamo Region

With Figure 2, we present a schematic diagram that illustrates in spherical coordinates the flow and magnetic field components for the proposed dynamo.

As seen from Eq. (3), and illustrated in Figure 2, the toroidal magnetic field, ΔB_ϕ , generated by ΔV_ϕ , would peak at low latitudes where the meridional gradient of the zonal flow is largest. This magnetic field component would tend to peak at the height (radial distance) where the flow peaks. Since ΔB_θ would change direction at this height, we are justified in ignoring the term $V_\phi \Delta B_\theta$ and indicate this with a square bracket.

In a similar vane, we can further simplify Eqs. (4) and (5). Applying Eq. (4) at the equator, the vertical (radial) component of the flow, ΔV_r , interacts with the given meridional magnetic field, B_θ , to generate ΔB_θ . Since ΔV_r tends to peak at the same height as ΔV_ϕ , where ΔB_θ changes direction, we may ignore the term with $V_r \Delta B_\theta$, as indicated with square bracket. In Eq. (5), the radial flow velocity, ΔV_r , interacts with B_θ to generate ΔB_r at mid to high latitudes. Since ΔB_r and ΔV_r are expected to peak at the same height, and ΔB_θ changes direction there, we are justified to ignore the term with $V_r \Delta B_\theta$ in square bracket.

With the above simplifications, the Eqs. (3) through (5) then reduce to

$$i\omega \Delta B_\phi \approx \frac{1}{r} \frac{\partial}{\partial \theta} \Delta V_\phi B_\theta \quad (6)$$

$$i\omega \Delta B_\theta \approx -\frac{\partial}{\partial r} \Delta V_r B_\theta \quad (7)$$

$$i\omega \Delta B_r \approx \frac{1}{r} \frac{\partial}{\partial \theta} \Delta V_r B_\theta \quad (8)$$

which describe in simplified form, and independent of each other, the toroidal magnetic field, ΔB_ϕ , as well as the meridional and radial components of the poloidal magnetic field, $(\Delta B_\theta, \Delta B_r)$. Equations (7) and (8) for the poloidal magnetic field satisfy $\text{div}(\Delta B_\theta, \Delta B_r) = 0$.

To evaluate Eqs. (6) and (7), we make the analytic ansatz that ΔV_ϕ , ΔV_r , ΔB_ϕ vary radially like $\cos((r-r_0) 2\pi/\lambda)$ in the range $-\lambda/4 < r-r_0 < \lambda/4$ as illustrated in Figure 2, where λ is the vertical wavelength of the flow oscillation taken to be 15 Mm (Mayr et al., 2001). The corresponding variations for ΔV_θ , ΔB_θ , are then $\sin((r-r_0) 2\pi/\lambda)$, which approximately assure that there is no net mass transport in the meridional direction and that the magnetic flux closes ($\text{div} B = 0$). The latitude dependence of ΔV_ϕ is taken to be $\sin((\theta-90)360/\Phi)$ in the range $-\Phi/4 < \theta-90 < \Phi/4$, with $\Phi = 80^\circ$, so that the flow oscillation is confined within 20° around the equator. As seen from Eq. (6), the magnetic field, ΔB_ϕ , would then have the form $\cos((\theta-90)360/\Phi)$, centered at 20° from the equator, but with opposite signs in the two hemispheres. It is understood that the estimates we shall provide represent the amplitudes of the above analytic functions.

With $\Delta V_\phi = 20$ m/s, $\omega = 2\pi/22$ years $\approx 10^{-8}$ s $^{-1}$, $r = 5 \times 10^8$ m, and $(1/r)\partial/\partial\theta = 4.5/r \approx 10^{-8}$, we obtain $\Delta B_\phi/B_\theta \approx -i\Delta V_\phi/(m/s) \approx -i20$. The oscillating toroidal magnetic field generated by the flow oscillation is thus much larger than the time independent meridional component of the poloidal magnetic field, which assures that this field component changes direction in 11 year intervals. And the magnetic field would peak when the zonal flow changes direction (factor i).

To estimate from Eq. (7) ΔB_θ of the poloidal magnetic field, we need to know the vertical (radial) flow component, ΔV_r , which cannot be related to the wave driven zonal flow, ΔV_ϕ , in a straight forward way. Unlike the zonal circulation at the equator, which can be derived simply from the zonal momentum equation, the vertical and meridional flow velocities that make up the meridional circulation require a 2D solution of the full set of conservation equations for energy, continuity and momentum. This difficulty notwithstanding, the following simplified scale-analysis serves to provide some insight.

Considering continuity of flow, the equations of momentum and energy conservation are written in the form

$$i\omega\Delta V_\phi + \Omega\Delta V_\theta + \frac{K}{h^2}\Delta V_\phi = MS(GW) \quad (\text{wave driven zonal momentum balance}) \quad (9)$$

$$\frac{gh}{LT} \Delta T = \Omega \Delta V_\phi \quad (\text{heliostrophic meridional momentum balance}) \quad (10)$$

$$\left(i\omega + \frac{\alpha}{c_p} \right) \Delta T \approx i\omega \Delta T = -S \Delta V_r = S \frac{h}{L} \Delta V_\theta \quad (\text{energy balance}) \quad (11)$$

with ΔT the temperature perturbation around T ; α the coefficient for Newtonian cooling; S the adiabatic temperature lapse rate or stability; c_p the specific heat at constant pressure; K the vertical eddy diffusivity; $h = \partial/\partial r$ and $L = (1/r)\partial/\partial\theta$ respectively the characteristic vertical and horizontal scales of the perturbation; Ω the mean rotation rate of the solar atmosphere; and $MS(GW)$ the gravity wave momentum source. For Eq. (11) we considered (C. L. Wolff, private communication) that the time constant for radiative transfer is long relative to 22 years in the dynamo region.

Eliminating ΔT in Eqs. (10) and (11), we obtain the meridional and radial flow velocities

$$\Delta V_\theta = \frac{i\omega L^2 T}{gh^2 S} \Omega \Delta V_\phi, \quad \Delta V_r = -\frac{i\omega LT}{gh S} \Omega \Delta V_\phi \quad (12)$$

in terms of the azimuthal (zonal) flow velocity.

As expected, the meridional flow, ΔV_θ , varies with the zonal flow, ΔV_ϕ , which is generated in our model by the wave interaction in Eq. (9). Substituting ΔV_r from Eq. (12) into Eq. (7), we obtain $\Delta B_\theta/B_\theta = [LT/(gh^2 S)] \Omega \Delta V_\phi$. The poloidal magnetic field thus reaches peak magnitudes around the time when the toroidal magnetic field changes sign, which is observed at the solar surface. Also, the oscillating poloidal magnetic field increases with Ω , consistent with the empirical scaling law advanced by Blackett (1947), similar to Bode's law, for massive astrophysical bodies that tend to reveal such a trend.

As pointed out earlier, without a 2D numerical solution for the flow field, it is difficult to evaluate $\Delta B_\theta/B_\theta$. Since ΔV_r varies inversely with S , and the convective stability approaches zero at the base of the convection region as seen from Figure 1a, a situation develops that is indeterminate. Moreover, $\Delta B_\theta/B_\theta$ varies with h^{-2} , and this scale parameter is not well known. In principle, ΔV_r thus may take on any value, and that value could perhaps be large enough to

produce in Eq. (7) a ratio $\Delta B_\theta/B_\theta \gg 1$, so that the poloidal magnetic field, like the toroidal field, reverses even in the dynamo region. A more conservative estimate of the poloidal field would suggest, based on Eq. (9), that the Coriolis acceleration from the meridional flow can at most be comparable to the zonal flow acceleration, which means that $\Omega \Delta V_\theta \approx i\omega \Delta V_\phi$. With $h = 15 \times 10^6 / 2\pi$ m, $L = 10^8$ m, $\Omega = 2.7 \times 10^{-6}$ s⁻¹, this translates for Eq. (7) into $\Delta B_\theta/B_\theta \approx -i\Delta V_\theta/(L\omega) \approx \Delta V_\phi/(L\Omega) \approx 0.1$. In the dynamo region, the oscillating poloidal magnetic field component, ΔB_θ , is probably small compared with the time independent component, B_θ , and therefore much smaller than the toroidal magnetic field, ΔB_ϕ .

The above analysis, applied to the dynamo source region, may provide the boundary conditions for realistic 3D models that describe magnetic field transport to the solar surface. Upward transport of ΔB_θ by advection (V_r term in Eq. (4) with square bracket) as well as convection and diffusion should be more efficient than the transport of the background magnetic field, B_θ , that presumably has a larger spatial size. In the convection region, in 3D, flux loops can form from the toroidal magnetic field to generate a poloidal field (α effect), as was demonstrated by Parker (1955, 1979) and confirmed with numerical dynamo models (see the reviews by Gilman, 2000; Coudhuri, 2000). One may expect then that $\Delta B_\theta/B_\theta$, at the surface, increases significantly over the value we estimated for the dynamo region, so that the poloidal magnetic field, like the toroidal magnetic field, would reverse in 11-year intervals, as observed.

4. Summary and Conclusions

Given an oscillating flow with a period of 22 years that would develop due to wave mean flow interaction in the stable layer of the Sun just below the convective envelope (Mayr et al., 2001; Mayr and Wolff, 2001), we discussed here some properties of the kinematic dynamo that may generate corresponding oscillations in the magnetic field. With the kinematic dynamo being linear in the magnetic field, we rely on the existence of a time independent magnetic field, \mathbf{B} , that would permit the flow oscillation, $\Delta \mathbf{V}$, to generate through the Lorentz force the magnetic field oscillation, $\Delta \mathbf{B}$. The analysis presented here was simplified in that it was restricted to the dynamo region. No attempt was made to explain the nature of the time independent magnetic field component. Also, we made no attempt to describe how the magnetic fields from the source

region would be transported by advection, convection and diffusion to the solar surface, which is of critical importance and requires a realistic 3D MHD model.

For a given wave driven zonal flow oscillation, understood to be confined to low latitudes, we employed scale-analysis to estimate the associated meridional circulation and the oscillating toroidal and poloidal magnetic fields that are generated. Our analysis leads to the following conclusions:

1) To first order, in the dynamo region, the oscillating toroidal field, ΔB_ϕ , and poloidal field component, ΔB_θ , depend mainly on the time independent field component, B_θ .

2) The toroidal magnetic field oscillation, ΔB_ϕ , is produced primarily, and directly, by the oscillating zonal flow. With the wave driven zonal flow velocity, ΔV_ϕ , being largest at the equator, ΔB_ϕ peaks at low latitudes where the latitudinal gradient of the flow is largest. The magnetic fields in the two hemispheres have opposite polarities in agreement with observations. The phasing is such that ΔB_ϕ peaks at the time when ΔV_ϕ changes direction. Relative to B_θ , the oscillating component, ΔB_ϕ , is large in the source region.

3) The poloidal magnetic field oscillation is produced by the oscillating meridional circulation. At the equator, ΔB_θ is related to B_θ and to the vertical gradient of the vertical flow velocity. In contrast to (2), this magnetic field component is in phase with ΔV_ϕ . Since the meridional and zonal circulations are out of phase, the toroidal magnetic field is largest during times when the poloidal magnetic field changes direction, which agrees with observations. The ratio $\Delta B_\theta/B_\theta$ is small in the source region.

4) Compared with the time independent component, B , the oscillating fields generated in the dynamo region, ΔB , are presumably transported more efficiently by advection, convection and diffusion to the solar surface. In the convection region, in 3D, flux loops can also form from the toroidal magnetic field to generate a poloidal field. We may expect then that the oscillations would dominate at the surface to produce the observed reversals in the toroidal and poloidal magnetic fields.

5) Our analysis shows that wave driven flow oscillations produce a meridional circulation and a related poloidal magnetic field that increases with the mean rotation rate of the fluid. This is consistent with the empirical scaling law of Blackett (1947), also called Bode's law, which reveals for massive astrophysical bodies that the magnetic moment tends to increase with the angular momentum.

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Figure Captions

Figure 1. (a) Buoyancy frequency, N , plotted versus relative radial distance to the center of the Sun. This is taken from a standard radiative transfer model. Since dynamics is not accounted for, such a model would overestimate N , and this is indicated with dashed line. (b) Azimuthal flow with peak velocity, $U = 20$ m/s, at low latitudes, which is assumed to be centered at $0.68 R$ in the tachocline and decaying flow below it. The pattern drifts slowly downward with a new flow developing at the top. This causes the flow at a fixed location to reverse about every 11 years. (c) Over time, the solar rotation profile (solid) should shift back and forth between the dashed curves (assuming that the solid profile applies to the time when $U = 0$). This would make the tachocline look more, or less, extended during the 22-year cycle. (Figure taken from Mayr and Wolff, 2001.)

Figure 2. Schematic that illustrates on the sphere the flow velocities, $\Delta \mathbf{V}(\omega t) = (\Delta V_\phi, \Delta V_\theta, \Delta V_r)$ and the related magnetic fields, $\Delta \mathbf{B}(\omega t) = (\Delta B_\phi, \Delta B_\theta, \Delta B_r)$. The toroidal field has opposite polarities in both hemispheres. Since the meridional and zonal (azimuthal) circulations are out of phase, the poloidal field peaks at the time when the toroidal field changes direction. The insert defines the functions that are employed in the simplified scale-analysis to describe the radial (height) variations of different variables.

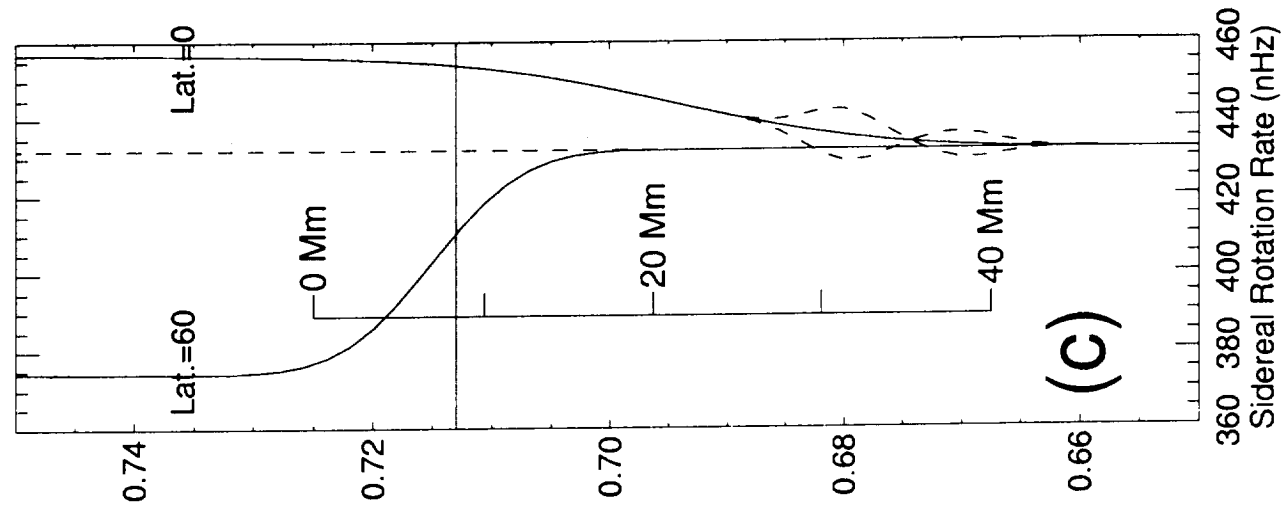
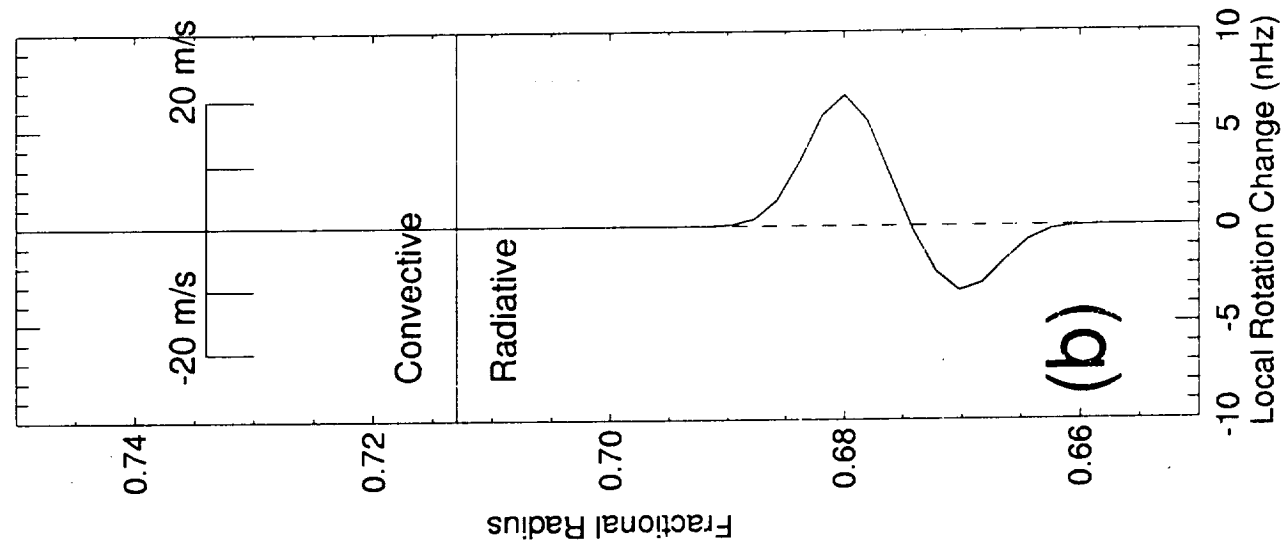
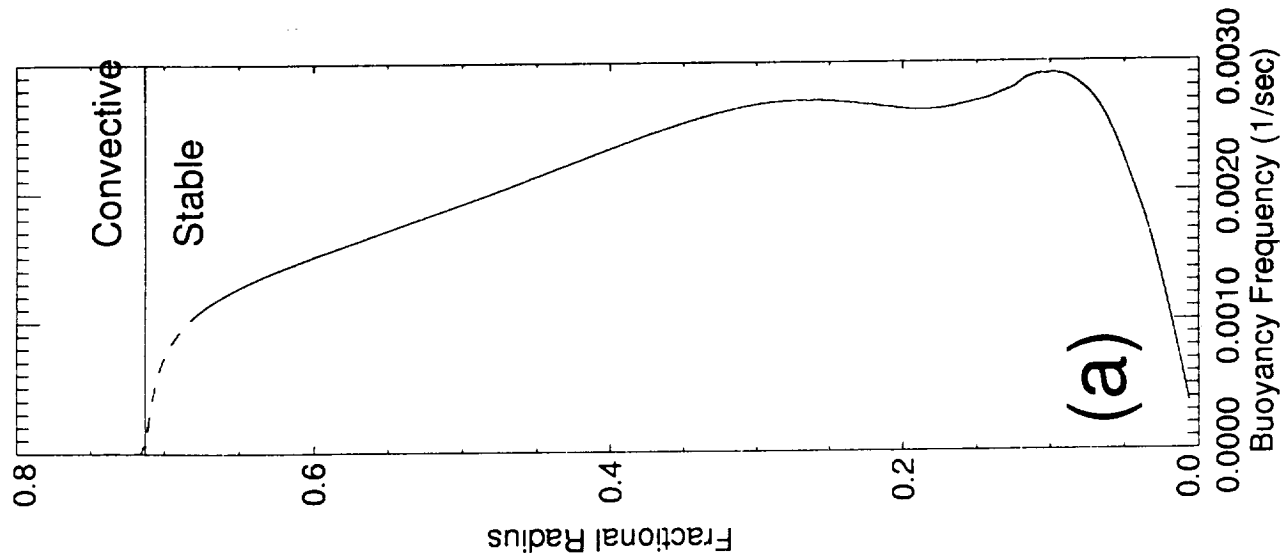


Figure 1

Figure 2 is a schematic diagram illustrating the geometry and vector fields in a tokamak cross-section. The main diagram shows a cross-section with a central column containing two regions labeled '+' and '-'. Various vector fields are indicated: ΔV_ϕ (toroidal flow), ΔV_θ (meridional flow), ΔV_r (radial flow), ΔB_ϕ (toroidal field), ΔB_θ (poloidal field), and ΔB_r (radial field). A dashed line represents the equator, and a solid line represents the pole. A small inset shows the spatial distribution of these fields, with ΔV_ϕ and ΔB_ϕ following a \cos distribution and ΔV_θ and ΔB_θ following a \sin distribution.